- f is <u>1-1</u> if for each y in Range(f), there is exactly one x in the Domain(f) that $f(x)=y$.

Even/odd
- f is even if $f(-x) = f(x)$ $\}$ for all x.
  f is odd if $f(-x) = -f(x)$

- f' = the instaneous rate of change of f
  = the slope of the tangent line to f.

- $f'>0 \Leftrightarrow f$ increasing nearby.
  $f'<0 \Leftrightarrow f$ decreasing nearby.
  $f'=0 \Leftrightarrow$ Stationary pt: f is not changing much nearby.

<u>Transform</u>
- $f(x)+c$: shift up by c units
  $f(x+c)$: shift right by $-c$ units.
  $k\,f(x)$: scale vertically by ratio k.
  $f(kx)$: scale horizontally by ratio $1/k$.
  $-f(x)$: vertical flip
  $f(-x)$: horizontal flip.

- $\lim_{x\to b} f(x) = L \Leftrightarrow \lim_{x\to b^-} f(x) = \lim_{x\to b^+} f(x) = L$.

<u>Continuous</u>
- f is continuous at $x=b$, if
  $\lim_{x\to b^-} f(x) = \lim_{x\to b^+} f(x) = f(b)$.

<u>IVT</u>
- Intermediate Value Theorem:
  f continuous on $[a,b]$, then all intermediate values between $f(a)$ and $f(b)$ are achieved.

<u>EVT</u>
- Extreme Value Theorem:
  f continuous on $[a,b]$, then there is a maximum and a minimum

<u>MVT</u>
- Mean Value Theorem:
  f continuous on $[a,b]$, differentiable on (a,b), then there is $c\in(a,b)$ that
  $$f'(c) = \frac{f(b)-f(a)}{b-a} = \text{average rate of change of } f \text{ between } a, b.$$

Limits:

- $\lim_{x\to b}(f(x) \pm g(x)) = \lim_{x\to b} f(x) \pm \lim_{x\to b} g(x)$.
- $\lim_{x\to b}(f(x)\,g(x)) = (\lim_{x\to b} f(x))(\lim_{x\to b} g(x))$.
- $\lim_{x\to b}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to b} f(x)}{\lim_{x\to b} g(x)}$.

- If h is continuous, $\lim_{x\to b} h(g(x)) = h(\lim_{x\to b} g(x))$.
- $f(x) \le g(x)$ then $\lim_{x\to b} f(x) \le \lim_{x\to b} g(x)$.

Sandwich

- Sandwich theorem: if $f(x) \le g(x) \le h(x)$, if $\lim_{x\to b} f(x) = \lim_{x\to b} = h(x) = L$, then $\lim_{x\to b} g(x) = L$.

Concavity

- First derivative test:
  - $\boxed{\cup}$ f' transitions $- \to +$ $\Rightarrow$ f concaves up. $\Leftrightarrow$ local min
  - $\boxed{\cap}$ f' transitions $+ \to -$ $\Rightarrow$ f concaves down. $\Leftrightarrow$ local max
  - $\boxed{\int}$ f' transitions $+ \to +$ or $- \to -$ $\Rightarrow$ no concavity. $\Leftrightarrow$ <u>NOT</u> local max/min

- Second derivative test:
  - $f'' > 0$ $\Rightarrow$ concave up $\Leftrightarrow$ local min
  - $f'' < 0$ $\Rightarrow$ concave down $\Leftrightarrow$ local max.
  - $f'' = 0$ $\Rightarrow$ <u>NO Conclusion</u>, use first derivative test instead.

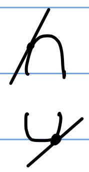
- Inflection point: $f'' = 0$ and f'' changes signs ($+ \to -$ or $- \to +$) through.

Optimization

- Critical point: ① Stationary pt, ② end pt, ③ where f' does not exist.

Approximation

- Tangent line approximation: fix point b,
  $$f(x) \approx f(b) + f'(b)(x-b).$$
- Overestimate if f concaves down
  Underestimate if f concaves up.



Derivative rules

- $(f+g)' = f' + g'$

  $(f-g)' = f' - g'$

  $(fg)' = f'g + fg'$

  $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$.

- $(x^k)' = k x^{k-1}$.

- Horizontal asymptote for rational functions:
  - $\frac{\text{small deg}}{\text{large deg}}$: $y = 0$.
  - $\frac{\text{equal deg}}{\text{equal deg}}$: $y = \frac{\text{leading coefficient}}{\text{leading coefficient}}$.
  - $\frac{\text{large deg}}{\text{small deg}}$: no horizontal asymptotes, $y \to \infty$ or $-\infty$ depending on situations.

- e^x grows faster than any x^k (any $k>0$) as $x \to \infty$, and $\ln(x)$ grows slower than any x^k (any $k>0$) as $x \to \infty$.